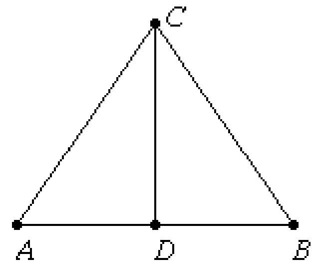
5-3: Isosceles and Equilateral Triangles

Theorem: **The Isosceles Triangle Theorem:** If two sides of a triangle are congruent, the angles opposite these sides are congruent.

Given: ΔABC with $\overline{AC} \cong \overline{BC}$

Prove: $\angle A \cong \angle B$



Plan: In order to prove this theorem, we will use the median to the base $\overline{AB}$ to separate the triangle into two congruent triangles.

Proof:

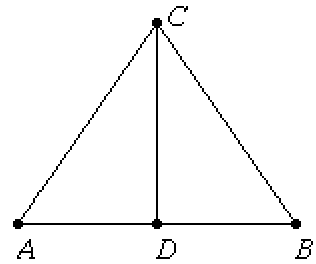
	Statements	***Reasons***
	1. Draw *D*, the midpoint of $\overline{AB}$	A line segment has one and only one midpoint
	2. $\overline{CD}$ is the median from *C*	Definition of median
S	3. $\overline{AD} \cong \overline{BD}$	
S	3. $\overline{AC} \cong \overline{BC}$	Given
S	4.	Reflexive Property
	5. $\Delta ACD \cong \Delta BCD$	
	$\therefore$	

- A **corollary** is a theorem that can be easily proven from another theorem. Two corollaries follow from this theorem.

Corollary: The median from the vertex angle of an isosceles triangle bisects the vertex angle.

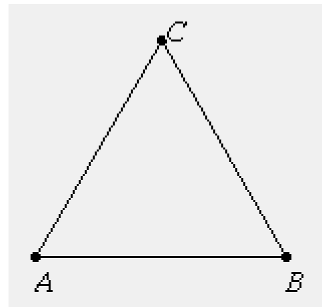
Proof: Since $\Delta ACD \cong \Delta BCD$, ________________________ by definition of congruent triangles. Thus $\overline{CD}$ divides $\angle ACB$ into two congruent angles, and is an angle bisector by definition.

Corollary: The median from the vertex angle of an isosceles triangle is perpendicular to the base.



Proof: Since $\Delta ACD \cong \Delta BCD$, $\angle ADC \cong \angle BDC$ by ___________________________ ___________________________. By theorem, if two lines intersect to form congruent angles, they are ______________________________. Thus $\overline{CD} \perp \overline{AB}$.

Corollary: Every equilateral triangle is equiangular.



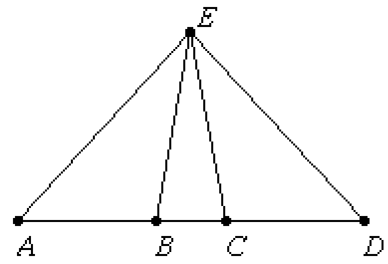
Proof: If ΔABC is equilateral, __________ $\cong$ __________ $\cong$ ___________. By the isosceles triangle theorem, if $\overline{AB} \cong \overline{BC}$ then ______________________, and if $\overline{BC} \cong \overline{CA}$, then _________________. Therefore, ________ $\cong$ ________ $\cong$ _________.

Example:

Given: E not on $\overline{ABCD}$, $\overline{AB} \cong \overline{CD}$,

$\overline{EB} \cong \overline{EC}$

Prove: $\overline{AE} \cong \overline{DE}$



Proof:

	Statements	***Reasons***
S	1. $\overline{EB} \cong \overline{EC}$	
	2.	Isosceles Triangle Theorem
	3. $\overline{ABCD}$	
	4. $\angle ABE$ and $\angle EBC$ are supplements; $\angle DCE$ and $\angle ECB$ are supplements	
	5. $\angle ABE \cong \angle DCE$	
S	6.	
	7. $\Delta ABE \cong \Delta DCE$	
	$\therefore$	Definition of Congruent Triangles

Homework: