8-5: Coordinate Proof

We can use the formula for slope, the slopes of perpendicular lines, and the coordinates of the midpoint of a line segment to prove theorems about triangles.

There are two types of proofs in coordinate geometry:

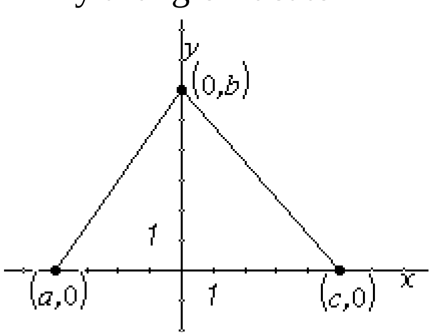
1. *Proofs Involving Special Cases*: When the coordinates of the endpoints of a segment or the vertices of a polygon are given as ordered pairs of numbers, we are proving something about a specific segment or polygon.
2. *Proofs of General Theorems*: When the given information is a figure that represents a particular type of polygon, we must state the coordinates of its vertices in general terms using variables. Those coordinates can be any convenient variables. Since it is possible to use a transformation that is an isometry to move a triangle without changing its size and shape, a geometric figure can be placed so that one of its sides is a segment on the x-axis. If two line segments or adjacent sides of a polygon are perpendicular, they can be represented as segments of the *x*-axis and the *y*-axis.

- To prove that line segments bisect each other, show that the coordinates of the midpoints are the same ordered pair, that is, the same point.

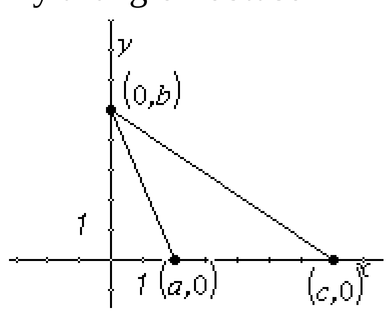
- To prove that two lines are perpendicular to each other, show that the slope of one line is the negative reciprocal of the other.

The vertices shown in these diagrams can be used when working with triangles:

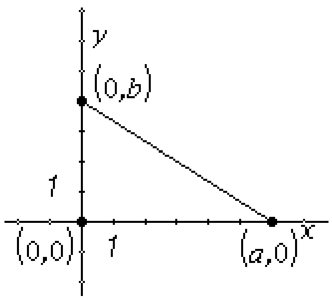
Any triangle - acute



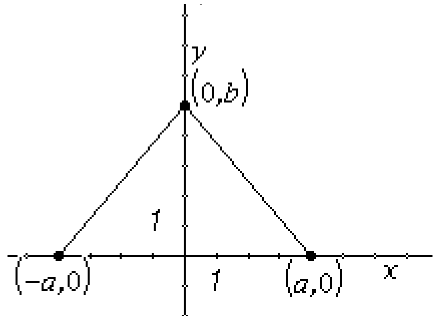
Any triangle - obtuse



Right triangle



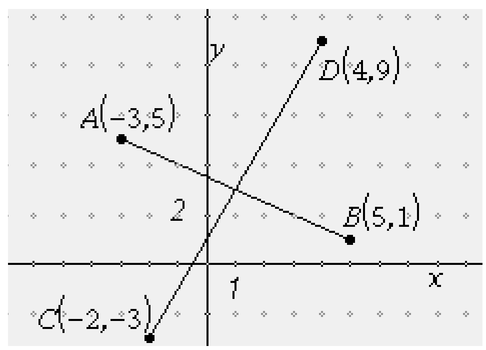
Isosceles triangle



- When **a general proof** involves the **midpoint** of a segment, it is helpful to express the coordinates of the endpoints of the segment as variables divisible by 2.

Example:

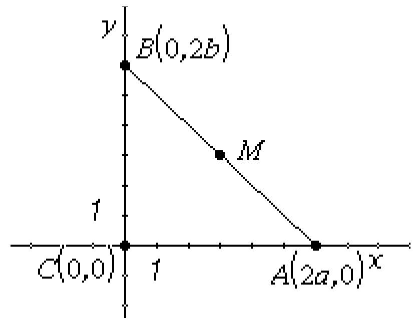
Prove that $\overline{AB}$ and $\overline{CD}$ bisect each other and are perpendicular to each other if the coordinates of the endpoints of the segments are *A*(-3, 5), *B*(5, 1), *C*(-2, -3), and *D*(4, 9). (Proof of a special case)



Example:
Prove that the midpoint of the hypotenuse of a right triangle is equidistant from the vertices.

- This is a proof of a general theorem. Since it is a right triangle, we can place one vertex at the origin, one side of the triangle on the *x*-axis, and a second side on the *y*-axis. We will use coordinates that are divisible by 2 to simplify computation of midpoints.

Given: Right triangle *ABC* whose vertices are $A(2a, 0)$, $B(0, 2b)$, and $C(0, 0)$. Let *M* be the midpoint of the hypotenuse, $\overline{AB}$.



Prove: $AM = BM = CM$

Proof:

Statements	***Reasons***
1. *M* is the midpoint of $\overline{AB}$	
2. $\overline{AM} \cong \overline{BM}$	
3. The coordinates of *M* are	Midpoint formula
4. From *M*, draw a vertical segment that intersects $\overline{AC}$ at *N*. The coordinates of *N* are	*N* is on the same vertical line as *M*, and the same horizontal line as *A* and *C*
5. The coordinates of the midpoint of $\overline{AC}$ are $\rightarrow N$ is the midpoint of $\overline{AC}$	Midpoint formula
6. $\overline{AN} \cong \overline{NC}$	

7. $\overline{MN} \perp \overline{AC}$	
8. $\angle ANM$ and $\angle CMN$ are right angles	
9. $\angle ANM \cong \angle CMN$	
10. $\Delta ANM \cong \Delta CMN$	
11. $\overline{AM} \cong \overline{CM}$	
12. $\overline{AM} \cong \overline{BM} \cong \overline{CM}$	
$\therefore AM = BM = CM$	

Homework: