

## 9-1: Proving Lines Parallel

**Definition:** **Parallel lines** are coplanar lines that have no points in common, or have all points in common and, therefore, coincide.

- For any two coplanar lines,  $\overline{AB}$  and  $\overline{CD}$  three possibilities exist:
  1.  $\overline{AB}$  and  $\overline{CD}$  have no points in common  $\rightarrow \overline{AB}$  is parallel to  $\overline{CD}$ .
  2.  $\overline{AB}$  and  $\overline{CD}$  have only one point in common  $\rightarrow \overline{AB}$  and  $\overline{CD}$  intersect.
  3.  $\overline{AB}$  and  $\overline{CD}$  have all points in common  $\rightarrow \overline{AB}$  and  $\overline{CD}$  are the same line.

Example:

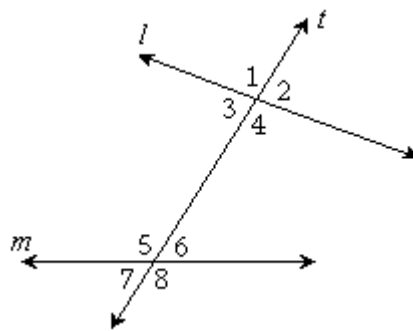
If line  $l$  is not parallel to line  $p$ , what statements can you make about these two lines?

### Parallel Lines and Transversals

**Definition:** A **transversal** is a line that intersects two other coplanar lines in two different points.

Two lines,  $l$  and  $m$ , are cut by transversal  $t$ . The angles formed can be classified:

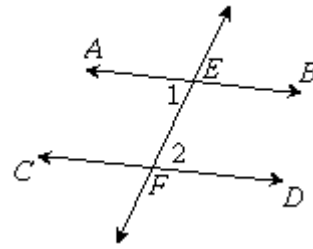
- **interior angles** are between  $l$  and  $m$
- **exterior angles** are not between  $l$  and  $m$
- **alternate interior angles** are on opposite sides of the transversal
- **alternate exterior angles** are on opposite sides of the transversal
- **interior angles on the same side of the transversal**
- **corresponding angles** are one interior and one exterior angle on the same side of the transversal



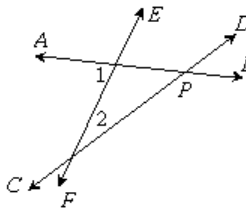
**Theorem:** If two coplanar lines are cut by a transversal so that the alternate interior angles formed are congruent, then the two lines are parallel.

*Given:*  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$  are cut by transversal  $\overleftrightarrow{EF}$   
at points  $E$  and  $F$ , respectively.  $\angle 1 \cong \angle 2$

*Prove:*  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



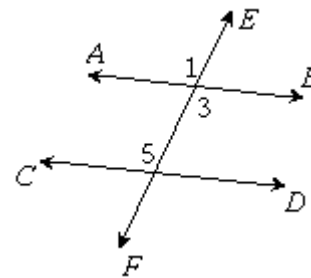
- We use an **indirect** proof.

<i>Proof:</i>	<i>Statements</i>	<i>Reasons</i>
	1. $\overleftrightarrow{AB}$ is <b>not</b> parallel to $\overleftrightarrow{CD}$	<b>Assumption</b>
	2. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ intersect at some point, $P$ , forming $\triangle EFP$	
		
	3. $\overleftrightarrow{AB}$ and $\overleftrightarrow{CD}$ are cut by transversal $\overleftrightarrow{EF}$ at points $E$ and $F$ , respectively	
	4. $m\angle 1 > m\angle 2$	
	5. But, $\angle 1 \cong \angle 2$	
	6. $m\angle 1 = m\angle 2$	
	$\therefore$	<b>Contradiction</b> in steps 4, 6

**Theorem:** If two coplanar lines are cut by a transversal so that the corresponding angles formed are congruent, then the two lines are parallel.

*Given:*  $\overleftrightarrow{EF}$  intersects  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ ,  $\angle 1 \cong \angle 5$

*Prove:*  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$

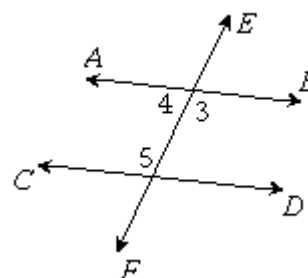


<i>Proof:</i>	<i>Statements</i>	<i>Reasons</i>
	1.	Given
	2. $\angle 1 \cong \angle 3$	
	3. $\angle 3 \cong \angle 5$	
	$\therefore$	

**Theorem:** If two coplanar lines are cut by a transversal so that the interior angles on the same side of the transversal are supplementary, then the two lines are parallel.

*Given:*  $\overleftrightarrow{EF}$  intersects  $\overleftrightarrow{AB}$  and  $\overleftrightarrow{CD}$ ,  $\angle 5$  is the supplement of  $\angle 4$

*Prove:*  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



*Proof:* Angle 3 and angle 4 are a linear pair, so they are \_\_\_\_\_.

If two angles are supplements of the same angle, then \_\_\_\_\_.

Therefore, since  $\angle 5$  is also the supplement of  $\angle 4$ , \_\_\_\_\_.

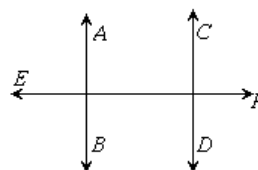
Angles 3 and 5 are a pair of congruent alternate interior angles. If two coplanar lines are cut by a transversal so that the alternate interior angles are congruent,

\_\_\_\_\_. Therefore, \_\_\_\_\_.

**Theorem:** If two coplanar lines are each perpendicular to the same line, then they are parallel.

*Given:*  $\overleftrightarrow{AB} \perp \overleftrightarrow{EF}$ ,  $\overleftrightarrow{CD} \perp \overleftrightarrow{EF}$

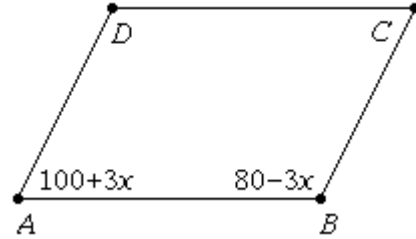
*Prove:*  $\overleftrightarrow{AB} \parallel \overleftrightarrow{CD}$



*Strategy:* Show that a pair of alternate interior angles is congruent.

Example:

If  $m\angle A = 100 + 3x$  and  $m\angle B = 80 - 3x$ , explain why  $\overline{AD} \parallel \overline{BC}$ .

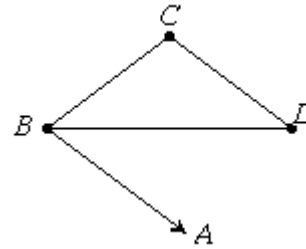


Example:

If  $\overline{BD}$  bisects  $\angle ABC$ , and  $\overline{BC} \cong \overline{CD}$ , prove  $\overline{CD} \parallel \overline{BA}$ .

Given:

Prove:



<i>Proof:</i>	<i>Statements</i>	<i>Reasons</i>
	1. $\overline{BC} \cong \overline{CD}$	
	2.	isosceles $\Delta \rightarrow$ base angles $\cong$
	3. $\overline{BD}$ bisects $\angle ABC$	
	4.	
	5.	Substitution
	6. $\angle DBA$ and $\angle D$ are alternate interior angles	
	$\therefore$	

Homework:

## Methods of Proving Lines Parallel

To prove that two coplanar lines that are cut by a transversal are parallel, prove that any one of the following is true:

1. A pair of alternate interior angles is congruent.
2. A pair of corresponding angles is congruent.
3. A pair of interior angles on the same side of the transversal is supplementary.
4. Both lines are perpendicular to the same line.

## Summary of Properties of Parallel Lines

If two lines are parallel:

1. A transversal forms congruent alternate interior angles.
2. A transversal forms congruent corresponding angles.
3. A transversal forms supplementary angles on the same side of the transversal.
4. A transversal perpendicular to one line is also perpendicular to the other.
5. A third line in the same plane that is parallel to one of the lines is also parallel to the other.

